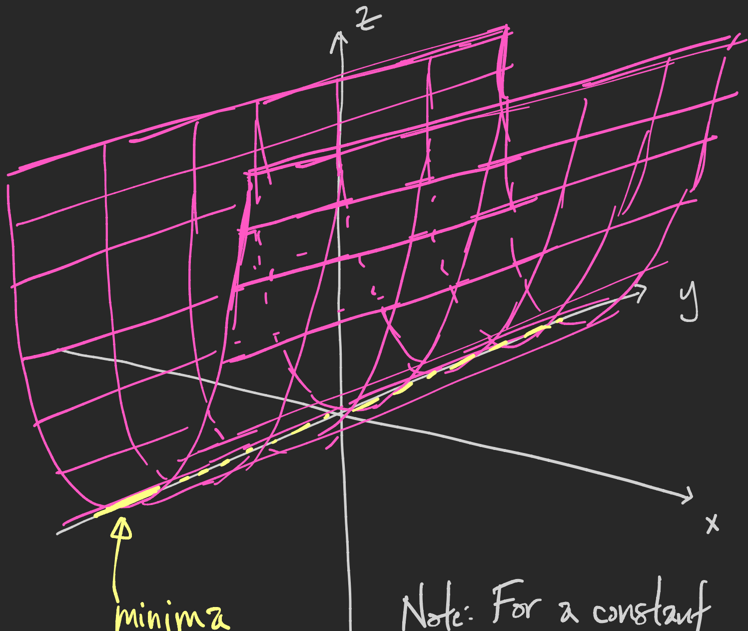


Note: For a constant function $g(x,y)=5$ for example, every point is a local/absolute max/min.

Let's analyze this algebraically:

① Identify the critical points: $\nabla f = \vec{0}$ (or the gradient DNE, but this doesn't happen here)

$$\begin{cases} 2x = 0 \\ 0 = 0 \end{cases}$$

So the critical points are all points of the form $(0, y)$.

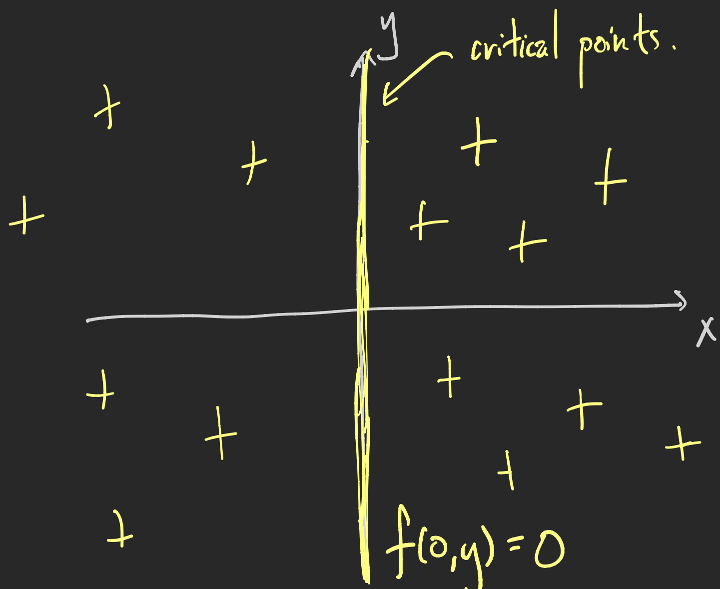
② Attempt to classify using 2nd derivative test.

$$H_f(x,y) = \begin{bmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

determinant $D = 0$ (at all points)

$\therefore$ 2nd deriv test is inconclusive.



Need to analyze function directly.

To understand whether (a,b) is an absolute min, ask:

"Is it the case that $f(x,y)$ is never strictly <u>smaller</u> than $f(a,b)$?"

YES $\longrightarrow$ (a,b) is abs. min ✓

NO $\longrightarrow$ (a,b) is not abs. min

To understand whether (a,b) is a local min, ask:

"Can I draw a circle around (a,b) such that $f(x,y)$ is never strictly smaller than $f(a,b)$ in that <u>circle</u>?"

YES $\longrightarrow$ (a,b) is local min

NO $\longrightarrow$ (a,b) is not a local min.

Example:

"$(0,2)$ is an absolute minimum of f because $\underbrace{f(x,y)}_{= x^2} \geq f(0,2) = 0$ for all x,y."